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A SLUG FLOW MODEL IN A LONG MILK TUBE FOR DESIGNING A MILKING UNIT CONTROL SYSTEM

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ABSTRACT

Automatic milking differs from natural sucking behavior, and it affects animal health. Subatmospheric pressure is the main parameter of a mechanical milker, and it is produced to overcome teat resistance, open the teat sphincter, keep the milking unit attached to teats, suck milk from the teat sinus and transport milk to the pipeline. Bernoulli's equation was used to minimize the effect of inflowing air on the pressure drop between the milking cluster and the pipeline. A diagram for calculating the reduction in pressure was developed using the Goal Seek tool in MS Excel. A minimum pressure drop was noted at every milk flow rate when inflowing air had the required flow rate. In the proposed control system, the area of the air inlet was adjusted to minimize the pressure drop during milking. The developed model can be used to improve automatic control solutions in a milking unit.

Key words: milking machine, automatic milking system, milk flow modeling, vacuum drop, flow rate

Abbreviations: A – cross-sectional area of the air inlet in the long milk tube; c – coefficient given by formula (15); D – diameter of the inlet in the long milk tube; E_G – average gas volume fraction in the mixture; Eo – Eötvös number; f – friction coefficient for liquid flow in the tube; g – acceleration of gravity, $g = 9.81$ m·s^{-2}; H – milk delivery head; k – local loss coefficient; L – length of the long milk tube; N_{LV} – liquid viscosity number; P – pressure; R – specific gas constant, for air R = 287.05 m^2·s^{-2} ·K^{-1}; T – absolute gas temperature; V – flow velocity; Q_G – standard volumetric gas (air) flow rate, m^3·h^{-1}; Q_m – volumetric milk flow rate, dm^3·min^{-1}; ρ – density; σ – surface tension of a liquid; μ – dynamic viscosity coefficient; ΔP – pressure drop; γ – isentropic exponent.

Subscripts: *amb* – ambient ; *b* – bubble; *bs* – bubble rise; *G* – gas (air); *m* – milk; *M* – mixture; *mean* – mean; *std* – standard reference conditions; *w* – operating parameters in the milk pipeline.

INTRODUCTION

The first mechanical milking machine was built more than 150 years ago to reduce the need for human labor during cow milking. This goal was achieved by mechanization, automation and, subsequently, by the use of milking robots. Technical equipment in barns is investigated to determine the most effective conditions for maximizing production in dairy farms. The above also applies to milking (Gaworski and Leola, 2015). Mechanical milking and the advancements made in this area significantly influence the functionality of raw materials in the dairy industry.

The invention of milking machines gave rise to new problems in the area of cow health, hygiene and well-being as well as milk quality. Mechanical milking differs considerably from the physiological mechanism of teat suction, and it affects the animals' health (Grinchenko *et al*., 2016; Jacobs and Siegford, 2012; Mačuhová *et al*., 2003; Mein, 2012; Mein and Reinemann, 2009; Stefanowska *et al*., 2000; Wenzel *et al*., 2003).

The suction force in a mechanical milking machine is produced by vacuum. The main technical parameter of a mechanical milking machine is subatmospheric pressure and its fluctuations (Luberański *et al*., 2013a, 2013b; Penry *et al*., 2016). Subatmospheric pressure is produced by a vacuum pump to overcome teat resistance, open the teat sphincter, keep the milking unit attached to teats, suck milk from the teat sinus and transport milk to the pipeline and the container. The pressure drops between the milk pipeline and teat-end chambers of the apparatus, as a result of which, the above processes are carried out less effectively. Atmospheric air is introduced to the milking unit to minimize the pressure drop and reduce pressure fluctuations.

Various solutions for controlling and automating the milking process have been proposed, including by

Ambord and Bruckmaier (2010), Armstrong and Daugherty (1997), Besier *et al.* (2016), Devir *et al.* (1996), Devir *et al.* (1997), Hansen (1999), Hillerton (1997), Hogeveen *et al.* (2001), Ipema and Hogewerf (2008), Kaihilahti *et al.* (2007), LeMire *et al.* (1999), Ordolff (2001), Reinemann *et al.* (2001, 2002~~, 2003, 2007~~), Rose-Meierhöfer *et al.* (2009). The economic aspects of mechanized milking have been discussed by Cooper and Parsons (1999). Therefore, milking is a highly complex process for a variety of reasons, and it requires a systemic approach (Kupczyk, 1999; Majkowska, 2009; Iwaszko *et al.*, 2012; Dróżdż *et al.*, 2018). Some authors analyzed the factors that affect the vacuum inside the milking claw by using a simulated milking device and by measuring milking claw vacuum when adjusting the flow rate (Enokidani *et al.*, 2016, 2017). Besier and Bruckmaier (2016) have investigated the effects of high system vacuum and extremely low claw vacuum during milk flow on milking performance and teat condition after milking.

The amount of air admitted to a mechanized milking unit has to be controlled, and the debate on the most effective air flow rates continues. According to Nordegren (1980), the milking process is most effective when the air flow rate equals 0.54 $m^3 \cdot h^{-1}$ (9 $dm^3 \cdot min^{-1}$), Kerkhoff (1972) proposed values in the range of 0.24 to 0.72 $m^3 \cdot h^{-1}$ (4–12 $dm^3 \cdot min^{-1}$), whereas Rossing (1970) set the maximum flow rate at 0.24 $m^3 \cdot h^{-1}$ (max. 4 $dm^3 \cdot min^{-1}$). The above values apply to cases where air is admitted to the system through an inlet with a constant cross-sectional area.

The aim of the milk production process is to obtain high-quality raw materials for further processing in a dairy plant. Milk quality is influenced by milking conditions, and mechanical milkers can compromise udder health through mechanical injury, backflow of milk which causes mastitis, and unstable pressure which contributes to the formation of free fatty acids (FFAs), in particular in high-line systems. It should also be noted that low-quality raw milk can decrease processing effectiveness in a dairy plant. This study proposes a method for stabilizing the pressure in a milking unit connected to a high-line system to avoid selected defects of milking machines equipped with pipelines. The aim of this study was to modify air flow reaching a milking unit, such as a cluster, to minimize the pressure drop between the pipeline and the cluster.

MATERIALS AND METHODS

System configuration: The discussed milking unit is presented in Figure 1.

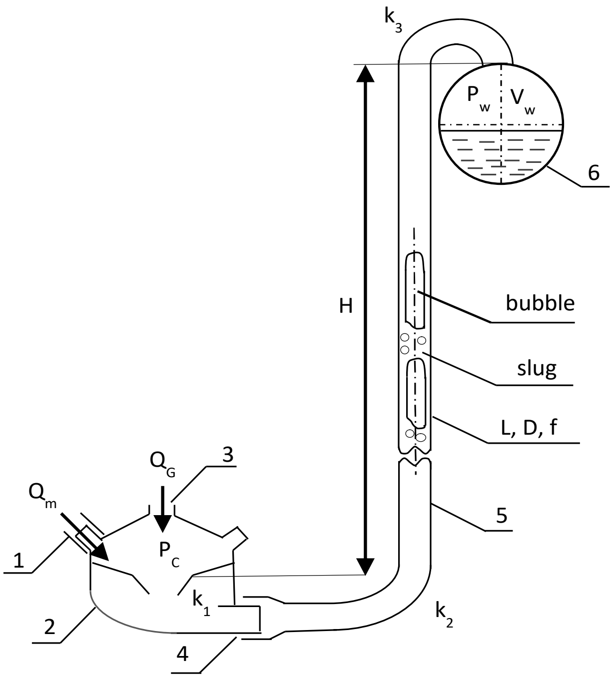

Figure 1. Milking unit: 1 – short milk tube, 2 – milking cluster, 3 – air inlet, 4 – outlet port, 5 – long milk tube, 6 – milk pipeline.

Milk flows from teats via the short milk tube 1 to milking cluster 2 which is supplied with ambient air via inlet 3. The air and milk mixture flows through outlet 4 and the long milk tube 5 to milk pipeline 6 and the container. In contemporary milking systems, teat cup shells in a milk cluster have a combined volume of 100 to several hundred cubic centimeters. Different solutions have also been proposed for milking machines, e.g. some milking robots are not equipped with milk clusters. The vacuum in the milk pipeline enables the milk to be transported from the cluster. The pressure drop between the milk pipeline and the teat-end chamber is a vital consideration in the milking process. In modern milking units with large clusters and wide milk tubes, pressure parameters registered in the teat-end chamber and the cluster are identical (O'Callaghan, 2004). This paper analyzes the drop in pressure between the milk pipeline and the cluster. Various authors have relied on Bernoulli's equation to describe the investigated phenomenon, among them Kupczyk (1999), Majkowska (2009) and Majkowska *et al.* (2008), but a comprehensive description of all processes observed in the analyzed section of the milk pipeline is not available in literature. Therefore, the objective of this study was to improve the mathematical description of the analyzed phenomena.

Problem formulation: Bernoulli's equation (1) was applied to two cross-sections separated by distance H (Figure 1) on the following assumption: the flow is steady and one-dimensional, and fluid is incompressible.

$$\frac{P_c}{\rho\, g} = H + \left(k_1 + k_2 + k_3 + f\frac{L}{D}\right)\frac{V_w{}^2}{2g} + \frac{V_w{}^2}{2g} + \frac{P_w}{\rho\, g} \qquad [1]$$

The terms on the right side of equation (1) represent: position head, head loss due to local resistance and friction, velocity head and pressure head. Friction loss in the tube and local losses are taken into account, whereas flow velocity in the cluster is disregarded.

The density of a flowing mixture $\rho = \left(1 - E_G\right)\rho_m + E_G\rho_G$ can be simplified to $\rho = \left(1 - E_G\right)\rho_m$ because milk is considerably more dense than air. The expressions $\Delta P = P_c - P_w$ and $k = k_1 + k_2 + k_3$ are introduced to equation (1) to produce formula (2):

$$\Delta P = \Delta P_1 + \Delta P_2 + \Delta P_3 \qquad [2]$$

where:

$$\Delta P_1 = \left(1 - E_G\right)H\rho_m g \qquad [3]$$

$$\Delta P_2 = \left(1 - E_G\right)\left(k + f\frac{L}{D}\right)\frac{V_w{}^2}{2}\rho_m \qquad [4]$$

$$\Delta P_3 = \left(1 - E_G\right)\frac{V_w{}^2}{2}\rho_m \qquad [5]$$

Pressure drop ΔP is calculated at given values of Q_G and Q_m when the remaining values in equation (1) are known. The slug flow theory (Govier and Aziz, 1972) was used to calculate E_G and V_w. The pressure drop between the pipeline and the cluster is numerically equal to the pressure drop between the cluster and the milk pipeline. The presented model can be used in milking machines in parlors equipped with both low-line and high-line pipelines, but it plays a more important role in milking machines connected to a high-line system (Fig. 1). The presented model has to be simplified for a milking machine connected to a low-line system.

Determination of mixture flow velocity: The average absolute velocity of the mixture is the sum of two components representing gas (air) and milk, respectively:

$$V_M \equiv V_w = V_G + V_m \qquad [6]$$

V_M can be calculated with the use of the formulas (7), (8), (9), 10) and (11) which are presented in successive parts of the article.

Gas velocity V_G is calculated as the ratio of volumetric flow rate of gas *in situ*, i.e. under local conditions in the long milk tube, to cross-sectional area A of the tube:

$$V_G = \frac{Q_{G\ in\ situ}}{A} \qquad [7]$$

$$A = \frac{\pi}{4}D^2 \qquad [8]$$

In practical applications, volumetric flow rate Q_G of gas flowing to the milking cluster is expressed in $m^3 \cdot h^{-1}$ under standard conditions.

$$Q_{G\ in\ situ} = \frac{Q_G}{3600}\frac{T}{T_{std}}\frac{P_{std}}{P} \qquad [9]$$

Pressure P in the long milk tube will be represented by the average of pressures P_c and P_w. Expression $\Delta P = P_c - P_w$ is introduced to $P_{mean} = \frac{1}{2}\left(P_c + P_w\right)$, and the result is:

$$P \equiv P_{mean} = P_w + \frac{\Delta P}{2} \qquad [10]$$

Milk flow velocity is calculated as the ratio of the volumetric flow rate of milk to cross-sectional area, as indicated above. In practical applications, the volumetric flow rate of milk is expressed in $dm^3 \cdot min^{-1}$, therefore:

$$V_m = \frac{Q_m \frac{0.001}{60}}{A} \qquad [11]$$

Determination of the average gas volume fraction: The average gas volume fraction is the average fraction of a vessel (cluster or long milk pipe) occupied by the liquid or gaseous phase in a unit of time. For example, if the liquid phase occupies around 20% of the vessel, the remaining part is occupied by gas. The average *in situ* gas volume fraction in the mixture is given by the below formula (Govier and Aziz, 1972):

$$E_G = \frac{V_w}{V_b} \qquad [12]$$

where:

V_w - milk flow velocity in a pipeline

V_b - rise velocity

Rise velocity V_b of the Taylor bubble is the sum of rise velocity V_{bs} in a stagnant fluid and C_0V_M proportional to mixture velocity. If the Reynolds number of the mixture is higher than 8000, then (Govier and Aziz, 1972) $C_0 = 1.2$ and:

$$V_b = V_{bs} + 1.2V_M \qquad [13]$$

The rise velocity of a bubble in a stagnant liquid is given by the following formula:

$$V_{bs} = c\sqrt{gD\frac{\rho_m - \rho_G}{\rho_m}} \qquad [14]$$

where coefficient c is equal to (Govier and Aziz, 1972):

$$c = 0.345\left[1-\exp\left(-\frac{0.01N_{LV}}{0.345}\right)\right]\left[1-\exp\frac{3.37-Eo}{m}\right] \quad [15]$$

the liquid viscosity number is

$$N_{LV} = \frac{1}{\mu_m^2}\sqrt{(\rho_m-\rho_G)\rho_m g D^3} \quad [16]$$

and the Eötvös number is

$$Eo = \frac{1}{\sigma}(\rho_m-\rho_G)\rho_m g \quad [17]$$

In equation (15), coefficient m takes the value of $m = 10$ if $N_{LV} > 250$, which represents a typical range of flow values in the long milk tube. The density of air in the long milk tube will be calculated from the ideal gas equation:

$$\rho_G = \frac{P_{mean}}{RT} \quad [18]$$

Based on the given formulas, we can conclude that ΔP is the only unknown variable in equation (2). Nevertheless, ΔP appears on both sides of this equation. The implicit form of the formula suggests that an iterative procedure is needed to solve it. The Goal Seek tool in MS Excel can be used for this purpose (Figure 2).

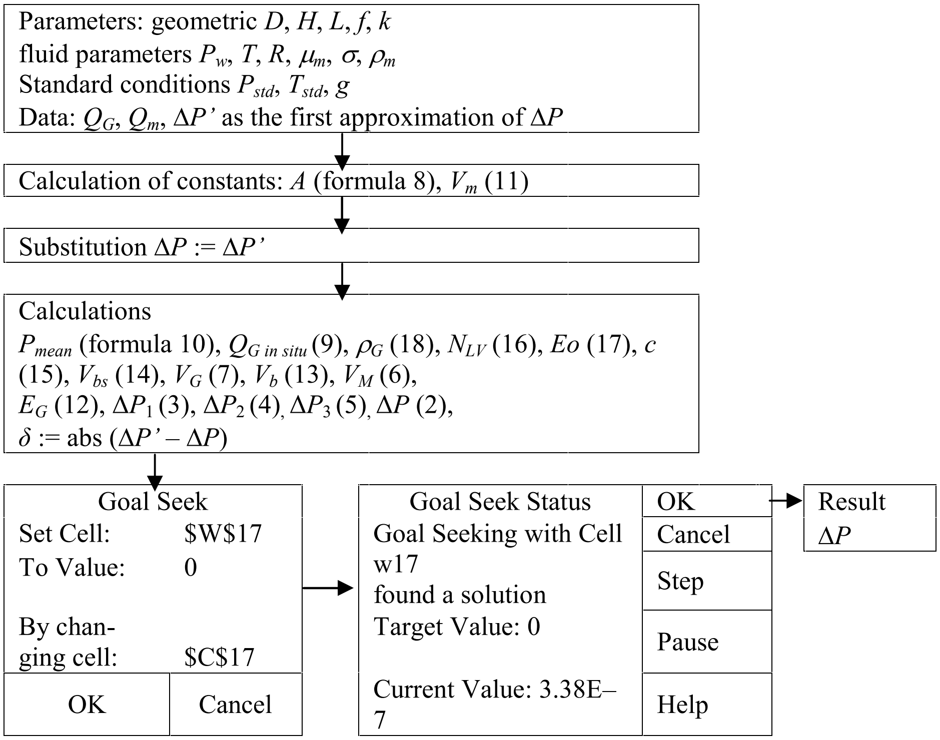

Figure 2. Block diagram of the process of calculating pressure drop ΔP with the Goal Seek tool in MS Excel. The first approximation of *ΔP'* was written in cell c17 and *δ* – in cell w17 in the same sheet.

In future research, the authors intend to analyze the stability of subatmospheric pressure in a milking machine with the use of the mathematical model presented by Kupczyk (1999) and Majkowska (2009).

RESULTS

The results were analyzed based on formulas (2), (3), (4) and (5).

The results calculated for an arbitrary value of Q_m = 5 dm^3·min^{-1} are shown in Figure 3. The remaining values were as follows: D = 0.013 m, H = 1.8 m, L = 2.1 m, f = 0.02 (the correlation between f, the Reynolds number Re and tube wall roughness was disregarded), k = 0.55 (tube inlet + 90º elbow + 180º elbow, according to Idelchik, 2008), P_w = 49.3 kPa, T = 303 K, μ = 0.0012 kg·s^{-1}·m^{-1}, (Scott and Reitsma, 1978), σ = 0.072 N·m^{-1}, ρ_m = 1030 kg·m^{-3}, P_{std} = 101325 Pa, T_{std} = 288.15 (U.S. Standard Atmosphere, 1976), g = 9.81 m·s^{-2}.

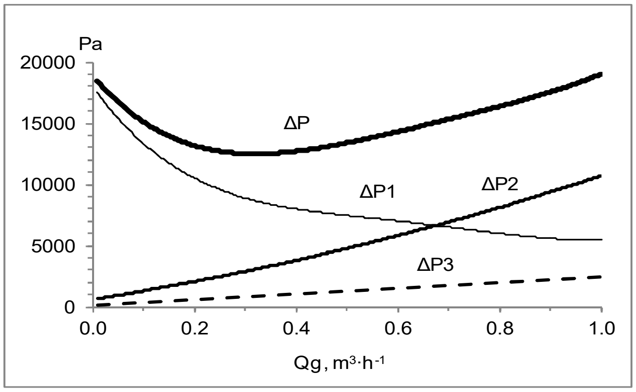

Figure 3. Total pressure (or vacuum) drop *ΔP* along the long milk tube and its components ΔP_1, ΔP_2, ΔP_3, calculated based on the block diagram in Fig. 2, as a function of air flow rate for Q_m=5 dm^3·min^{-1}.

The dynamic viscosity coefficient μ of milk was adopted from a relatively old source, but changes in its value have a negligible effect on calculation results.

The pressure values calculated for the above data and, additionally, for ambient pressure p_{amb} = 100 kPa, are shown in Figure 4.

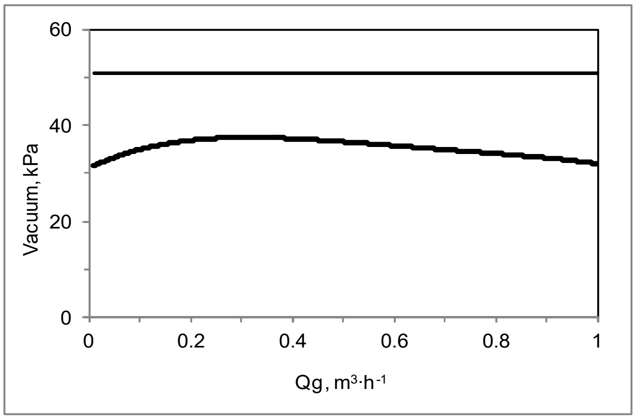

Figure 4. Vacuum in the milk pipeline (top line) and the milking cluster (bottom line) of a milking unit as a function of air flow rate, calculated for Q_m = 5 dm^3·min^{-1}; p_{amb} = 100 kPa.

Total pressure drop ΔP as a function of air flow rate has a minimum value, as demonstrated in Fig. 3. Under the measured conditions, the smallest drop in pressure ΔP was noted when Q_G ranged from 0.2 to 0.4 m^3·h^{-1}. The existence of a minimum can be attributed to the following: an increase in Q_G decreases the value of term (1 – E_G) in equations (3)…(5), but it also increases flow velocity in equations (4) and (5). When those effects are balanced out, the minimum value is obtained.

An interesting practical problem can be solved when the set of computational tools is expanded to include the MS Excel Solver. Our aim is to find the value of Q_G which, for every value of Q_m, minimizes pressure drop *ΔP*. The solution to this problem is shown in Figure 5.

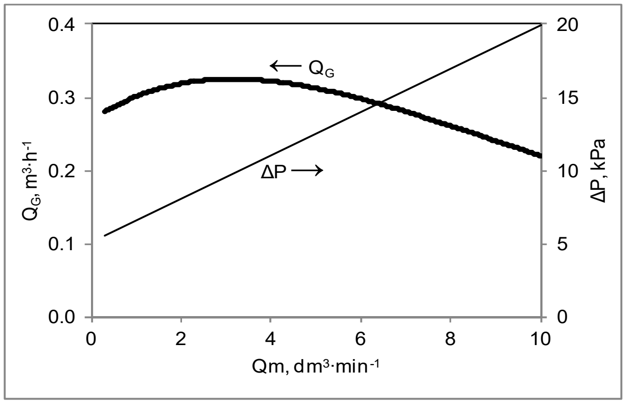

Figure 5. Air flow rate Q_G which minimizes the pressure (vacuum) drop, and the values of Δ*P* as a function of the milk flow rate.

The existence of minima suggests that the cross-sectional area of the air inlet in the milking cluster can be adjusted to minimize the overall pressure drop during the entire milking process which is characterized by fluctuations in milk flow rates.

In future research, the proposed model will be verified based on the findings of Kupczyk (1999) and Majkowska (2009).

Control concept: The proposed control concept is explained with an example. Let us assume that milking velocity as a (dimensionless) function of time can be approximated by three straight line segments, as shown in Figure 6a. Dimensionless time is defined as the ratio of real time to total milking time. The tree segments were connected at arbitrary points with coordinates (0.3, 10 dm^3·min^{-1}) and (0.8, 5 dm^3·min^{-1}). Terminal points were neglected because two-phase flow does not take place at the beginning or the end of milking.

Let us calculate the cross-sectional area of the air inlet which minimizes the pressure drop. The mass flow rate of gas flowing through an inlet with cross-sectional area A_d can be calculated using a modified Saint-Venant and Wantzel formula (Brower, 1999):

$$\dot{m} = C_d A_d \frac{p_{amb}}{\sqrt{T_{amb}}} \sqrt{\frac{2\gamma}{(\gamma - 1)R}}\, \varphi(\varepsilon) \tag{19}$$

where function φ(ε) of pressure ratio ε is determined by flow conditions which can be choked or subcritical:

$$\varphi(\varepsilon) = \text{if } 0 \le \varepsilon < \varepsilon_* \text{ then} \varphi(\varepsilon_*) \text{ else} \varphi(\varepsilon) = \sqrt{\varepsilon^{\frac{2}{\gamma}} - \varepsilon^{\frac{\gamma+1}{\gamma}}} \tag{20}$$

When air reaches the milking cluster, the pressure ratio is equal to:

$$\varepsilon = \frac{P_w + \Delta P}{P_{amb}} \qquad [21]$$

Since the isentropic exponent for air is equal to $\gamma = 1.4$, the critical pressure ratio is $\varepsilon_* = 0.5283$ and function $\varphi(\varepsilon_*) = 0.2588$. The mass flow rate in $\mathrm{kg \cdot s^{-1}}$ can be calculated from the below formula:

$$\dot{m} = \frac{Q_G P_{std}}{3600\, R T_{std}} \qquad [22]$$

When (20)…(22) are substituted into (19), A_d is the only unknown variable. The cross-sectional area of the air inlet is shown in $\mathrm{mm^2}$ in the top curve in Figure 6b for an arbitrary value of coefficient $C_d = 0.8$. The ratio of the largest to the smallest area A_d is approximately 1.4, and it is not determined by C_d. A needle valve for controlling air flow to the milking cluster can be easily designed. The valve would be connected to the air inlet (item 3 in Figure 1), and it would be controlled by a proportional magnet. Two control signals would be sent to the electromagnet: one proportional to the pressure in the milk cluster and one proportional to the milk flow rate.

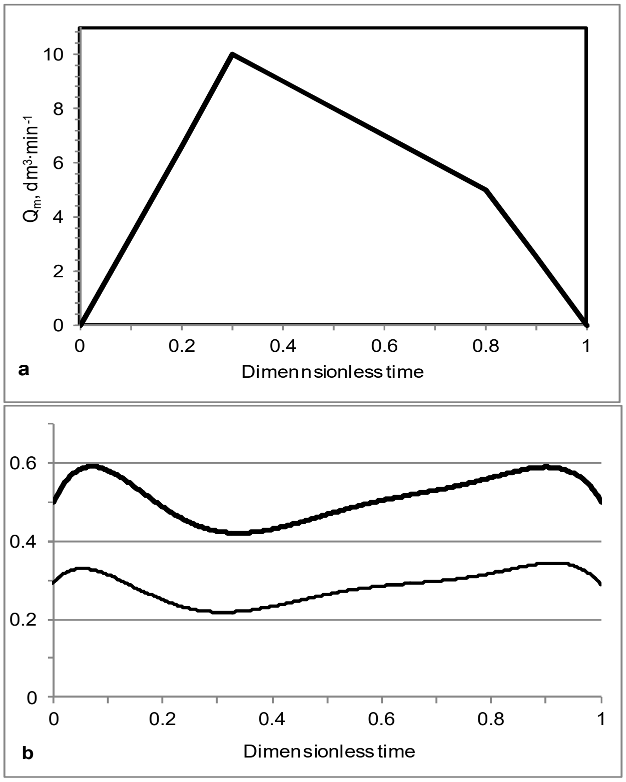

Figure 6. a) Milking velocity as a function of time, b) bottom curve - air flow rate that minimizes the pressure drop throughout the entire milking process, top curve – cross-sectional area A_d of the air inlet in the milking cluster.

DISCUSSION

Besier and Bruckmaier (2016) demonstrated that the minimum claw vacuum had the main influence on milking performance independent of the level of the system vacuum and related vacuum drops and a low minimum claw vacuum caused low milk flow and long milking times. Teat condition at the end of milking, however, was mainly dependent on the system vacuum, and the load on the teat tissue was obviously increased at a system vacuum of 50 kPa. As authors suggest this effect was obviously occurring toward the end of milking when milk flow decreased and hence the milk flow dependent vacuum drop disappeared (Besier and Bruckmaier, 2016).

The implementation of automatic milking systems in dairy production has caused an increase in lipolysis of bulk milk. Free fatty acids accumulated in milk result in the appearance of rancid flavours in dairy products (Wiking *et al.*, 2003).

O’Brien *et al.* (1998) suggest that elevated FFA levels in milk can impair the flavor quality and shelf life of milk and milk products and thus have implications for the dairy industry. An increase in milk FFA is induced when the protective fat globule membrane is disrupted and the fat exposed to the action of lipoprotein lipase. Mechanical abuse of milk in many modern milking systems arises from air admission to the milking system, height of milklines and pump operation. O’Brien *et al.* (1998) also claim that some of the machine factors contributing to excessive FFA development in milk can be eliminated by proper design, installation, maintenance and operation.

Wiking *et al.* (2003) have found that the milk fat globules (MFGs) were significantly larger in the milk with the highest fat content. An increase was found in FFAs for milk with the largest MFGs, indicating that milk with a high fat content is more unstable when exposed to mechanical stress. The mechanical stress that affects milk in the milking systems is a crucial factor in relation to milk quality (Wiking *et al.*, 2003). Wiking *et al.* (2006) have found that increased milking frequency increased milk yield, but fat content and fat yield were not affected. In automatic milking, the mechanical treatment of the milk is harsher compared with older systems, due to portions from each individual milking being continuously pumped, temperature fluctuation in the bulk tank and longer distances from the milking unit to the bulk tank. These effects cause an increase in lipolysis. Another factor contributing to the damage of MFGs is air. Uptake of air in milk will further increase the damage of MFGs imposed by mechanical treatments (Wiking *et al.*, 2003).

The discussed results were obtained for an exemplary data set, but the proposed method can also be used in systems with different geometric and flow parameters.

Conclusions: The pressure parameters associated with milking affect udder health, the speed and quality of the milking process, and the quality of obtained milk. This article analyzes the pressure parameters associated with the flow of a milk and air mixture in the long milk tube of a milking unit based on a two-dimensional flow pattern with slug structure. Bernoulli's equation was used to demonstrate that the pressure drop between the milking cluster and the milk pipeline has three components: position head, head loss due to local flow resistance and velocity head.

A high pressure drop is observed at low rates of air flow into the milking cluster. In the analyzed case, the milk and air mixture flows slowly, but it is characterized by high density. At higher air flow rates, the resulting decrease in pressure is also considerable due to the high velocity of the flowing mixture which, in the described case, is characterized by lower density. The pressure drop is minimized at a given air flow rate. In this study, pressure drop was minimized when the volumetric flow rate of gas ranged from 0.4 to 0.2 $\mathrm{m}^3\cdot\mathrm{h}^{-1}$ and the volumetric flow rate of milk ranged from 2 to 4 $\mathrm{dm}^3\cdot\mathrm{min}^{-1}$. This paper proposed a method for adjusting the cross-sectional area of the air inlet and controlling air flow rate to minimize the drop in pressure at all milk flow rate values. The proposed model can be used to improve the automatic control of the milking process. Due to the complexity of the mathematical model, the resulting equations cannot be solved analytically for any boundary and initial conditions. Therefore, numerical procedures have to be applied to produce quantitative results. The procedures implemented e.g. in the MS Excel Solver tool can be used to simulate the system's behavior under various external conditions and for different sets of internal parameters. Further research is planned to carry out more detailed numerical analyses and, in particular, to optimize the operation of the milking unit. The planned study will also more thoroughly investigate the effect of the applied approximations on the reported results. Milking equipment and milking conditions affect the content of FFAs in milk, but these considerations were not analyzed in this study and require separate research.

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