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TRANSFORMATION TO SOME GROWTH MODELS WIDELY USED IN AGRICULTURE

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ABSTRACT

A mathematical model is a tool used to obtain information about the behavior of a system. The mathematical models can be used to have a preliminary knowledge about the functioning of a system, reducing the product costs and improving the performance. The aim of this study is related to how some of the sigmoidal models widely used in agriculture are converted to mechanistic models which have biological meaning. For this purpose, the commonly used six different models with sigmoidal structure were used. These models are Gompertz, Janoschek, Richards, Stannard, Weibull and Vonbertalanffy models, respectively. The three parameters “maximum specific growth rate (μ_{max}), reached maximum value (A) and lag time (λ)” which are important in defining the growth, are shown, step by step, how to integrate into the models. As a result of this, these three biologically meaningful parameters have been gained in the models. In addition, we have also given on time of the inflection point and time of the inflection value of these models.

Keywords: Emprical model, Mechanistic model, Sigmoidal curve, Growth models, Lag time, Maximum specific growth rate, Reached maximum value

INTRODUCTION

A mathematical model is a powerful tool used to get valuable knowledge about the behavior of a system by using a mathematical technique. However, the main purpose of using the mathematical models is to obtain a preliminary knowledge about the functioning of a system, reducing the product costs and improving the performance (Tedeschi *et al*., 2005; Karkack, 2006). Mathematical models can be classified into several categories (Tedeschi *et al*., 2005). However, the mathematical models are collected under two classes: emprical and mechanistic models (Lopez *et al*., 2004; Uckardes, 2010). Emprical models contain with the parameters a, b, c, etc. Therefore, these models do not give an idea about the behavior of system, but they are used only to fit data points. Therefore, since the parameters of the emprical models do not contain any biological meaning. it is difficult to estimate their initial values. Whereas, mechanistic models describe the behavior of a system that also contain the parameters with biological meaning A, μ_{max}, λ, etc. Therefore, it is easy to estimate the initial values of the parameters of Mechanistic model and it is possible to calculate in 95% confidence intervals. Therefore, the researchers prefer mechanistic models in their studies (Lopez *et al*., 2004; Zwitering *et al*., 1990). Mathematical models have been widely prefferred in many disciplines such as economy, biology, chemistry and agriculture. Depending on the type of curve, the models used in these areas could be in forms which are linear, exponential, sigmoidal and etc. In agriculture, one of the widely used models is the model with sigmoidal structure. Sigmoidal models are considerably used in the studies of plant, poultry, ruminant, bacterial and fish growths. The aim of this study is to discuss to how some of the sigmoidal models widely used in agriculture are converted to mechanistic models which has biological meaning. In addition, more information on some new parameters of the models, namely, maximum specific growth rate, reached maximum value, lag time, time of the inflection point, its value and etc were also given.

Experimental data constructed in a theoretical framework are not given in this study. Akbas (1995) showed how the important parameters mentioned above in terms of growth for some growth models such as Logistic and Gompertz models may be obtained, but the equations of these parameters have not been integrated into the model. The aim of this study shows mathematically how the new parameters with biological meaning of the widely used six different sigmoidal growth models which have emprical structures can be integrated into the models. As a result, new mechanistic models that explain well the functioning of the system are obtained.

METHODS

Sigmodail curves show a similar shape to the letter “*S*” as depicted in Figure 1. A sigmoidal curve can be described under three-phases: starting phase, the phase of the rate and maximum phase.

Various models are used to describe these three-phases. A model having empirical form can define the curve very well, however it does not give any idea about the three phases. To define these three-phases, the form of mechanistic model should be created by adding several parameters to empirical models using mathematical transformations (Baranyi *et al*., 1994). As a result, these three phases can be determined. It is possible to define these three phases containing three parameters: the maximum specific growth rate, μ_{max}, defined as the tangent in the inflection point, the lag time, λ, defined as the x-axis intercept of this tangent and the asymptote A equal to a, defined as the maximum value reached.

There are many kinds of sigmoidal models as empirical form. Six of the commonly models are given in Table 1.

The following steps of the modification of these six models: Gompertz, Janoschek, Richards, Stannard, Weibull and Vonbertalanffy are given;

Step 1. To obtain the inflection point of the curve, the first and the second derivatives of the function with respect to t are given in Table 2 and Table 3, respectively.

Step 2. When the second derivative is equal to zero, time of the inflection point is found in Table 4. After this inflection point, the growth rates are decreased. It is important for the researchers to find this point.

Step 3. Now the maximum specific growth rate can be derived by substituting the inflection point in Table 4 in the first derivative in Table 2.

Step 4. The tangent line through the inflection point is is found by the formula

$$y = y(t_i) + \mu_{max}(t - t_i)$$

The lag time, λ, is defined as the t-axis intercept of the tangent through the inflection point. So, the equation below is written.

$$0 = y(t_i) + \mu_{max}(\lambda - t_i) \qquad (1)$$

By using Table 4 and Table 5, Equation 1 yields lag time in Table 6.

Step 5. In Table 7, the parameters in the models used can be substituted for the parameters of the mechanistic models.

Step 6. The asymptotic value is reached as t approaches infinity.

$$t \to \infty;\ y \to a \Rightarrow A = a$$

In all models, the parameter *a* can be substituted by A. So, the models yield the mechanistic forms of models in Table 8.

RESULTS

The partial derivatives of the models are presented in Tables 2 and 3. In Tables 4, 5, 6 and 7 the formulas of the time and value of the inflection point, the maximum growth rate, the lag time and meaningful parameters of models were given, respectively. The mechanistic models of the six empirical models mentioned above are given in Table 8.

As a result of the transition from Table 1 to Table 8, there is no change in the number of the parameters and the general structure of the models. Only the parameters of the models have changed. Therefore, there is no any change in the coefficient of determination of the models.

DISCUSSION

The first and second partial derivatives of the models are given in Tables 2 and 3, respectively. The first and second partial derivatives are necessary not only to convert the model from the empirical form into mecanistic form, but also to find the values of parameters of the model in the algorithm of some computer programs (Korkmaz *et al*., 2011). In Tables 4, 5, 6 and 7 the formulas of the time and value of the inflection point, the maximum growth rate, the lag time and meaningful parameters of models were given. Many authors such as Akbas and Oguz (1998); Narinc *et al*. (2010) described the time and values of the inflection point in different forms of the models of Gompertz and Richard in their studies. In addition, Sezer and Tarhan (2005) conducted a similar study for the model of Richard to calculate the time and values of the inflection point in their studies.

The mechanistic models of the studied six empirical models are given in Table 8. Similarly, to improve the effectiveness of the models Zwitering *et al*. (1990) and France *et al*. (2005) gave transformations from empirical models to mechanistic models in their studies. The model of Gompertz seems to be similar with modeling of the growth of bacteria done by Zwitering *et al*. (1990).

The formulas of some mechanistic models were much longer than the formulas of the empirical models (Table 8). Since all calculating processes for finding the parameters are made by using the package programs, the major factor limiting the use of a model is not related to the length of the model. It is related to the number of parameters (Motulsky and Ransnas, 1987; Zwitering *et al*., 1990; Khamis *et al*., 2005). Modified equation of each model seems like a long equation, but number of parameters in each equation just varied in the range of 3 to 5. The parameters of each model are biologically important to understand what they mean. Indeed, it is very important to know the meanings of the parameters in each model. When the meanings of the parameters are known, the model can easily fit in the data set. The models which have fewer number of parameters are easy to fit compared with the models with more number of parameters. Zwitering *et al*. (1990) emphasized that in case of the model which has fewer number of parameters to define the data set, this model should be preferred to the model with more number of parameters. The reasons for this are that the models which has more number of parameters are difficult to fit compared with the model with fewer number of parameters and the parameters of the models which has more number of parameters have some correlations with each. However, France *et al*., 2005 reported that the models which have fewer number of parameters may not give enough information about system. This situation should not be ignored. Therefore, When selecting a model, the researchers should take into account how much of the system is explained by the model and how the model agrees with the system. To do this, many researchers advice some statistical tests and criteria such as Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), Mean Square Error (MSE), Determination coefficient (R^2), Accuracy Factor (AF) and F-ratio test (Tedeschi *et al*., 2005; Lopez *et al*. 2004; Uckardes, 2010; Motulsky and Ransnas, 1987; Karakus *et al*., 2010 ; Tariq *et al*., 2011). The comparison of the models is not given in this study. These criteria will help the researchers to model selection.

Some researchers generally do not know if the studied model is mechanistic or empirical. In this study, the recognition of this distinction has been mentioned. Moreover, the transformations from some of the important sigmoidal emprical models to mechanistic models were gradually given. Furthermore; during these transformations, the first and second partial derivatives, the time of inflection point and its value, maximum specific growth rate, maximum value reached and lag time were also given in tables for all models.

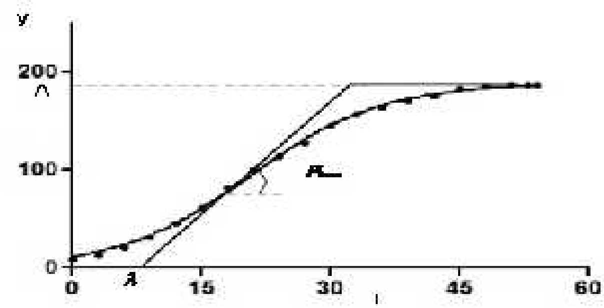

Figure 1. A Sigmoidal curve

Table 1. The models used as emprical form

Models	**Equation**
1. Gompertz	y=aexp(-exp(b-ct))
2. Janoschek	y=a-(a-b)exp(-ct^d)
3. Richards	y=a(1+bexp(-kt))^m
4. Stannard	y=a(1+exp(-(l+(kt))/m))^(-m)
5. Weibull	y=a-bexp(-kt^m)
6. Vonbertalanffy	y=a+(b-a)(1-exp(-ct))^(1/v)

Table 2. The first derivative of the models used in the study

Models	The First derivative (dy/dt)
1. Gompertz	acexp(b-ct)exp(-exp(b-ct))
2. Janoschek	(a-b)ct^ddexp(-ct^d)/t
3. Richards	-a(1+bexp(-kt))^mmbkexp(-kt)/(1+bexp(-kt))
4. Stannard	a(1+exp(-(l+kt)/m))^(-m)kexp(-(l+kt)/m)/(1+exp(-(l+kt)/m))
5. Weibull	bkt^mmexp(-kt^m)/t
6. Vonbertalanffy	(b-a)(1-exp(-ct))^(1/v)cexp (-ct)/(v(1-exp(-ct)))

Table 3. The second derivative of the models used in the study

Models	**The Second derivative (d^2y/dt^2)**
1. Gompertz	-ac^2exp(b-ct)exp(-exp(b-ct))+ac^2exp(b-ct)^2exp(-exp(b-ct))
2. Janoschek	(a-b)ct^dd^2*exp(-ct^d)/(t^2)-(a-b)ct^ddexp(-ct^d)/(t^2)-(a- b)c^2(t^d)^2d^2exp(-ct^d)/(t^2)
3. Richards	a(1+bexp(-kt))^mm^2b^2k^2exp(-kt)^2/((1+bexp(-kt))^2)+a(1+bexp(-kt))^mmbk^2exp(-kt)/(1+bexp(-kt))-a(1+bexp(-kt))^mmb^2k^2exp(-kt)^2/((1+bexp(-kt))^2)
4. Stannard	ak^2exp(-(l+kt)/m)((1+exp(-(l+kt)/m))^(-m-2)exp(-(l+kt)/m)m-(1+exp(-(l+kt)/m))^(-m-1)+(1+exp(-(l+kt)/m))^(-m-2)exp(-(l+kt)/m))/m
5. Weibull	bkt^mm^2exp(-kt^m)/(t^2)-bkt^mmexp(-kt^m)/(t^2)- bk^2(t^m)^2m^2exp(-kt^m)/(t^2)
6. Vonbertalanffy	(b-a)(1-exp(-ct))^(1/v)c^2exp(-ct)^2/(v^2(1-exp(-ct))^2)-(b-a)(1-exp(- ct))^(1/v)c^2exp(-ct) /(v(1-exp(-ct)))-(b-a)(1-exp(-ct))^(1/v)c^2exp(-ct)^2/(v(1-exp(-ct))^2)

Table 4. Time and value of the inflection points of the models used in the study

Models	**Time of the inflection point (t_i)**	**The value of the inflection point $y(t_i)$**
1. Gompertz	b/c	aexp(-1)
2. Janoschek	((d-1)/(cd))^(1/d)	a-(a-b)exp(-c(((d-1)/(cd))^(1/d))^d)
3. Richards	-ln(-1/(mb))/k	a(1-1/m)^m
4. Stannard	-(l-ln(m)m)/k	a(1+1/m)^(-m)
5. Weibull	((m-1)/(km))^(1/m)	a-bexp(-k(((m-1)/(km))^(1/m))^m)
6. Vonbertalanffy	-ln(v)/c	a+(b-a)(1-v)^(1/v)

Table 5. The maximum specific growth rate of the models used in the study

Models	Maximum specific growth rate (μ_{max})
1. Gompertz	acexp(-1)
2. Janoschek	(a-b)c(((d-1)/(cd))^(1/d))^dd((d-1)/(cd))^(-1/d)exp(-c(((d-1)/(cd))^(1/d))^d)
3. Richards	a((m-1)/m)^mkm/(m-1)
4. Stannard	a((m+1)/m)^(-m)k/(m+1)
5. Weibull	bk(((m-1)/(km))^(1/m))^mm((m-1)/(km))^(-1/m)exp(-k(((m-1)/(km))^(1/m))^m)
6. Vonbertalanffy	(b-a)(1-v)^(1/v)c/(1-v)

Table 6. Lag time of the models used in the study

Models	Lag time (λ)
1. Gompertz	(b-1)/c
2. Janoschek	-((d-1)/(cd))^(1/d)(exp(c(((d-1)/(cd))^(1/d))^d)(((d-1)/(cd))^(1/d))^(-d)a-(((d-1)/(cd))^(1/d))^(-d)a+(((d-1)/(cd))^(1/d))^(-d)b-cda+cdb)/((a-b)cd)
3. Richards	-(m-1+mln(-1/(mb)))/(mk)
4. Stannard	(-m-1-l+ln(m)m)/k
5. Weibull	-((m-1)/(km))^(1/m)(exp(k(((m-1)/(km))^(1/m))^m)(((m-1)/(km))^(1/m))^(-m)a-exp(k(((m-1)/(km))^(1/m))^m-(((m-1)exp(-m(mln(((m-1)/(km))^(1/m))-ln((m-1)/(km))))/(km))^(1/m))^mk(((m-1)/(km))^(1/m))^(m^2)((m-1)/(km))^(-m))(((m-1)/(km))^(1/m))^(-m)b-bkm)/(bkm)
6. Vonbertalanffy	-(-(1-v)^(-1/v)a+(1-v)^(-1/v)av-b+bv+a-av-bln(v)+aln(v))/((-b+a)c)

Table 7. The meaningful transformations of parameters used in models

Models	The meaningful transformations of parameters used in models	
1. Gompertz	b=1+qw/(aexp(-1))	c=w/(aexp(-1))
2. Janoschek	b=(-a-wq+exp(-(d-1)/d)da)/(exp(-(d-1)/d)d)	c=(d-1)((d-1)(a-b)/w)^(-d)exp(d-1)/d
3. Richards	b=-exp((m-1)(a+((m-1)/m)^(-m)qw)/(am))/m	k=w(m-1)/(a((m-1)/m)^mm)
4. Stannard	k=w(m+1)((m+1)/m)^m/a	l= (-am-a-qw((m+1)/m)^mm-qw((m+1)/m)^m+aln(m)m)/a
5. Weibull	b=exp((m-1+ln((m-1)(a+wq)/(wm))m)/m)w/(m-1)	k=(m-1)(b(m-1)/w)^(-m)exp(m-1)/m
6. Vonbertalanffy	b=(-a(1-v)^(-1/v)+a(1-v)^(-1/v)v+a-av-qw(1-v)^(-1/v)+qw(1-v)^(-1/v)v+ln(v)a)/(1-v+ln(v))	c=w(-1+v)(1-v)^(-1/v)/(-(-a(1-v)^(-1/v)+a(1-v)^(-1/v)v+a-av-qw(1-v)^(-1/v)+qw(1-v)^(-1/v)v+ln(v)a)/(1-v+ln(v))+a)

q= λ , w= μ_{max}

Table 8. Mechanistic equations of the models used in the study

Models	Mechanistic Equations
1. Gompertz	y=Aexp(-exp((A+qwexp(1)-wexp(1)t)/A))
2. Janoschek	y=-(-dA+exp(-(d-1)(((d-1)exp((d-1)/d)(A+wq)/(wd))^(-d)exp(d-1)t^d-1)/d)A+exp(-(d-1)(((d-1)exp((d-1)/d)(A+wq)/(wd))^(-d)exp(d-1)t^d-1)/d)wq)/d
3. Richards	y=A(-(-m+exp((m-1)(A+((m-1)/m)^(-m)qw-w((m-1)/m)^(-m)t)/(Am)))/m)^m
4. Stannard	y=A(1+exp(-(-Am-A-qw((m+1)/m)^mm-qw((m+1)/m)^m+Aln(m)m+w((m+1)/m)^mtm+w((m+1)/m)^mt)/(Am)))^(-m)
5. Weibull	y=(Am-A-exp(-(-m+1-ln((m-1)(A+wq)/(wm))m+exp((m-1+ln((m-1)(A+wq)/(wm))m)/m)^(-m)exp(m-1)t^mm-exp((m-1+ln((m-1)(A+wq)/(wm))m)/m)^(-m)exp(m-1)t^m)/m)w)/(m-1)
6. Vonbertalanffy	y=-(A-Av+ln(v)A-((exp(twv/(A+qw))-exp(tw/(A+qw))v^(tw/(A+qw)))exp(-twv/(A+qw)))^(1/v)A(1-v)^(-1/v)+((exp(twv/(A+qw))-exp(tw/(A+qw))v^(tw/(A+qw)))exp(-twv/(A+qw)))^(1/v)A(1-v)^(-1/v)v-((exp(twv/(A+qw))-exp(tw/(A+qw))v^(tw/(A+qw)))exp(-twv/(A+qw)))^(1/v)qw(1-v)^(-1/v)+((exp(twv/(A+qw))-exp(tw/(A+qw))v^(tw/(A+qw)))exp(-twv/(A+qw)))^(1/v)qw(1-v)^(-1/v)v)/(-1+v-ln(v))

q= λ , w= μ_{max}, t=time , A= the maximum value reached, exp=exponential, d,m,v=some parameters

Conclusions: In this study, it is shown step by step how to obtain the new models with the parameters which have biological meaning from the commonly used six different sigmoidal growth models which have empirical structures. More information about the growth with the parameters of the new models will be obtained. Experimental data has not been employed since this work has been prepared in a theoretical manner. In other study, the similarities and differences of the new forms of the models can be investigated by using experimental data.

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